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(Course Coordinator: W. Sarjeant)

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HIGH VOLTAGE/PULSE POWER TECHNOLOGY

GRADUATE COURSE EECS596

University of New Mexico

LECTURE INDEX

PREFACE

With the recent increase in technological needs and the interest in the power conditioning arena, one of the problems facing workers in the field is the lack of texts or notes describing recent progress, particularly in the area of repetitive power conditioning. For this reason and because of expanding internal requirements, the University of New Mexico (UNM) and the Los Alamos Scientific Laboratory (LASL) have created a set of lecture notes based upon the graduate course taught recently at UNM. The objective of these notes is to create a record of many of the advances in the field since the last text in the field was published just after World War II. In this context, the lectures presented are oriented toward an introduction of the reader to each of the areas described and present sufficient background information to explain many of these advances. They are not intended to serve as design engineering notes, and thus the reader is referred to the references at the end of each lecture for detailed technical information in specific areas.

The preparation of these writings is a result of a considerable teamwork effort on the part of LASL and Sandia staff. In particular, Cathy Correll, in conjunction with Jo Ann Barnes and the rest of her efficient word processing staff, carried the major responsibility for preparation of the lectures while the lecturers did the proofreading and revisions. As course coordinator, it is a pleasure to acknowledge the strong support of Ray Gore, our E-Division Leader, and Shyam Gurbaxani who is the UNM Graduate Center Director, Los Alamos Campus.

W. J. Sarjeant
Los Alamos Scientific Laboratory
Los Alamos, New Mexico
October 3, 1980

LECTURE 4

TRANSMISSION LINES AND PULSE FORMING NETWORKS

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Lecture 4

TRANSMISSION LINES AND CAPACITORS

by

R. R. Butcher

TRANSMISSION LINES AND PULSE FORMING NETWORKS

Bob Butcher

The topic of this lecture is pulse forming networks. The first item of discussion will be transmission lines because they are so prevalent, even if only in the form of coaxial cable. From there the subject will proceed to pulse-forming networks: the practical problems encountered with them, their advantages, and disadvantages. Capacitors will be our final topic, as they are the limiting factor in lumped transmission elements.

I have referenced two masters theses, and there is a bibliography available from Texas Tech University.[1,2] These present a good description of the transmission-line theory and more detail than I can present in the limited space here.

Fig. 1 shows the most familiar type of transmission line — coaxial cable. By notation, there is an inner radius A and an outer radius B (which is actually the outer radius of the inner conductor and the inner radius of the outer conductor and the inner radius of the outer conductor). This is an important point in that surfaces nearest each other are used because high-frequency currents flow near the surface.

Formulas for capacitance and inductance of coaxial transmission lines are:

$$C = \frac{2\pi\varepsilon_0\varepsilon_r}{\ln\frac{b}{a}} = \frac{5.56\times10^{11}\varepsilon_r}{\ln\frac{b}{a}} \quad \frac{F}{M} \qquad (1)$$

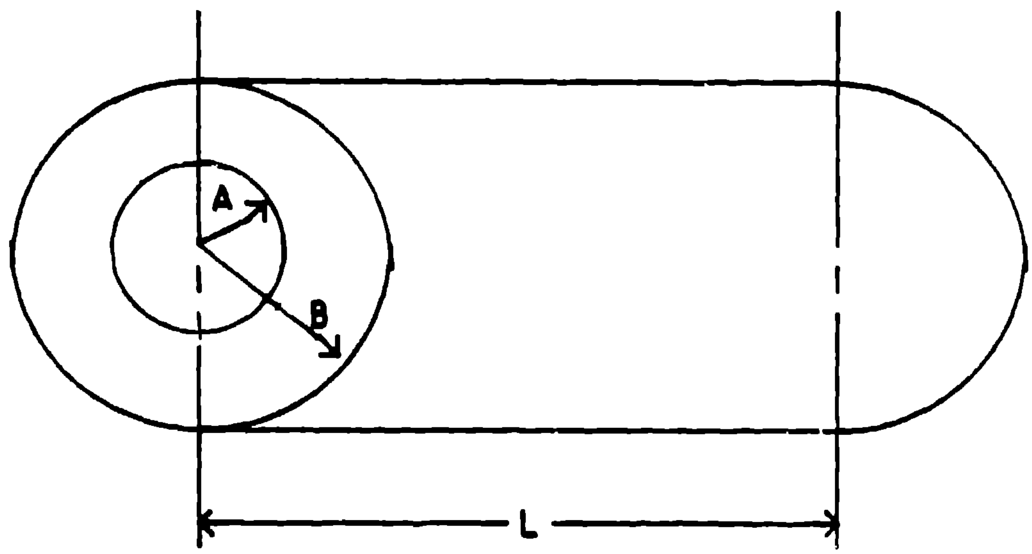


FIG 1: COAXIAL TRANSMISSION LINE

$$L = \frac{\mu_o \mu_r \ln \frac{b}{a}}{2\pi} = 2x10^{-7} \mu_r \ln \frac{b}{a} \qquad \frac{H}{m} \tag{2}$$

in units of Farads and Henries per meter of length. Since most high-voltage transmission lines do not use ferrite or other magnetic material as a filler, the value of μ_r is usually taken as 1, and it will be in the rest of this lecture. Given the inductance per unit length and the capacitance per unit length of a transmission line, the impedance of the transmission line can then be calculated.

$$Z_o = \sqrt{\frac{L}{C}} \tag{3a}$$

When this is reduced, it becomes:

$$Z_o = \frac{377}{2\pi\varepsilon_r} \ln \frac{b}{a} \qquad \text{ohms} \tag{3b}$$

where 377 ohms is recognized as the impedance of free space. The relative dielectric constant is sometimes referred to as K, although occasionally the total dielectric constant is simply lumped as ε. I generally tend to break this down as ε_o times ε_r, where ε_o is the permittivity of free space.

The one-way transit time of a transmission line is calculated by:

$$\tau_{1\text{-way}} = \sqrt{LC} = \sqrt{\mu_o \varepsilon_o \varepsilon_r} = \frac{\sqrt{\varepsilon_r}}{C} \qquad \frac{s}{m} \tag{4}$$

This is the time it takes the pulse to travel one meter along the transmission line. Notice that this reduces to the square root of the relative dielectric constant divided by the speed of light (C) and has units of seconds per meter.

If the total inductance and the total capacitance of the transmission line are known, one can find the total one-way propagation time of that line, or it can be done on a per-meter basis. Usually the numbers in tables are given on a per-foot or per-meter basis, and the impedance is a dimensionless quantity. The impedance does not depend on the length of the line: the length is assumed to be infinite.

The second common type of transmission line is the strip transmission line, which is characterized by two parallel plates (Fig. 2). This can be thought of as a coaxial cable that has been unfolded and laid out flat. In general, the thickness (t) is assumed to be much less than the width (w) of the line. The formulas for inductance and capacitance of the strip transmission lines are:

$$L = \mu_o \frac{t}{w} = 4\pi \times 10^{-7} \frac{t}{w} \qquad \frac{H}{m} \tag{5}$$

$$C = \varepsilon_o \varepsilon_r \frac{w}{t} = 8.854\text{x}10^{-12} \varepsilon_r \frac{w}{t} \qquad \frac{F}{m} \tag{6}$$

The impedance of the transmission line is:

$$Z_o = \sqrt{\frac{L}{C}} = \frac{377}{\sqrt{\varepsilon_r}} \frac{t}{w} \qquad \text{ohms} \tag{7}$$

one-way transmission time is again as it was in the coaxial line:

$$\tau_{1\text{-way}} = \sqrt{LC} = \mu_o \varepsilon_o \varepsilon_r = \frac{\sqrt{\varepsilon_r}}{C} \qquad \frac{s}{m} \tag{8}$$

A good approximation for impedance, if your thickness is <u>not</u> much less than the width is:

$$Z_o \simeq \frac{377}{\sqrt{\varepsilon_r}} \frac{t}{w} \, || \, \frac{377}{\sqrt{\varepsilon_r}} \tag{9}$$

where || denotes parallel impedance. It is obvious that this type of transmission line precludes impedances greater than 377 ohms. Even the 377-ohm level is difficult to attain if the dielectric material used is other than vacuum or air. Twin-lead line at 300 ohms is the highest value generally available.

Fig. 3 shows a battery switched onto the end of an uncharged transmission line. This transmission line is characterized by impedance Z_1 and a one-way transit time τ, and is terminated in an impedance Z_2. When the battery is switched into the line, a current of magnitude I_1 flows to the right. The arrows on the transmission line indicate that under certain conditions a reflected voltage and current wave, which are denoted V_1' and I_1' occur. These travel in the opposite direction down the transmission line (to the left in Fig. 3). At $t = 0$, when the switch is closed, a current of magnitude:

$$I_1 = \frac{V_1}{Z_1} \tag{10}$$

begins to flow to the right in the line. When this wave reaches the end of the line (at time $t = \tau$), the voltage and current at the end of the transmission line must be continuous. The voltage and current are considered to be the algebraic sum of the wave traveling to the right and the wave traveling to the left. This can be written as:

$$V_2 = V_1 + V_1' \tag{11}$$

where V_2 is the voltage across Z_2. Currents can be calculated in the same way, resulting in:

$$I_2 = I_1 + I_1' \tag{12}$$

Ohm's Law declares:

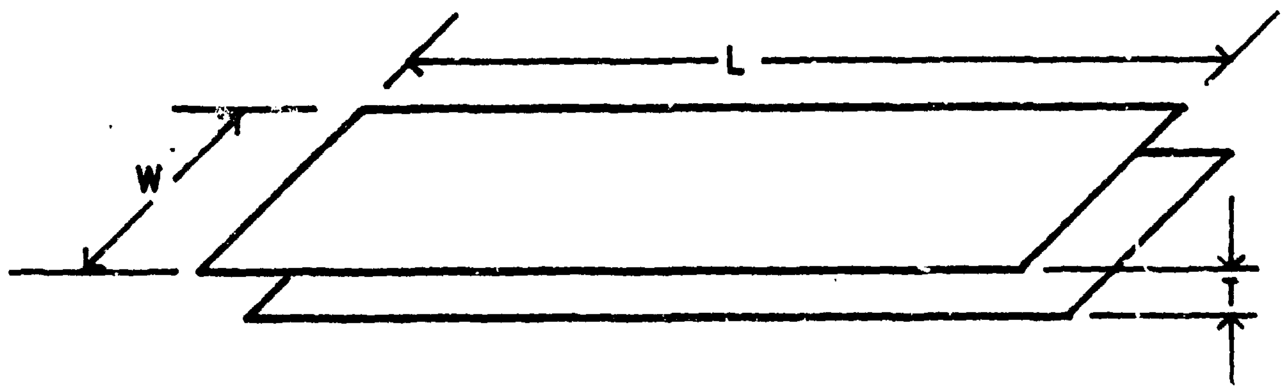


FIG 2: STRIP TRANSMISSION LINE

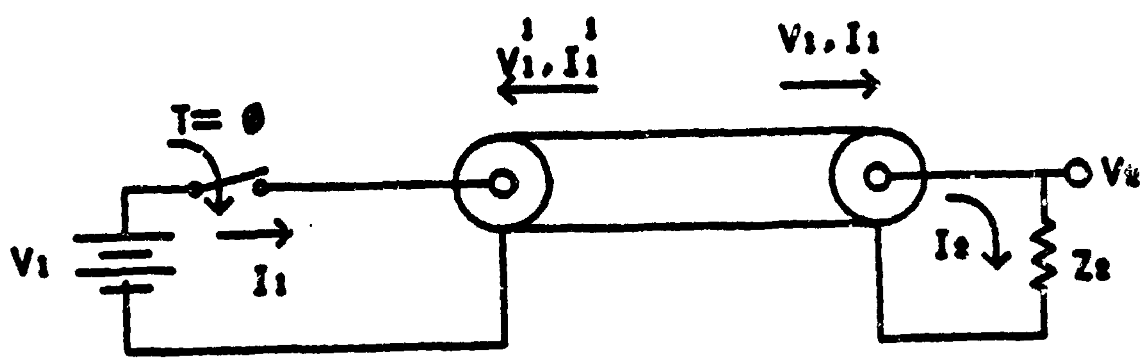


FIG 3: VOLTAGE & CURRENT WAVES IN TRANSMISSION LINE

$$I_2 = \frac{V_2}{Z_2} \tag{13}$$

We can also show that the current traveling to the left is:

$$I_1' = -\frac{V_1'}{Z_1} \tag{14}$$

which is negative by convention, due to the direction of travel. If equations (10), (12), and (14) are combined:

$$V_2 = \frac{V_1}{Z_1} - \frac{V_1'}{Z_1} \tag{15}$$

Combining equations (11) and (15) and defining

$$K = \frac{Z_2}{Z_1} \tag{16}$$

allows derivation of the voltage reflection coefficient

$$\frac{V_1'}{V_1} = \frac{Z_2 - Z_1}{Z_2 + Z_1} = \frac{1 - K}{1 + K} \tag{18}$$

The ratio of load current to line current, then, is

$$\frac{I_2}{I_1} = \frac{2Z_1}{Z_1 + Z_2} = \frac{2}{K + 1} \tag{19}$$

It is interesting to notice the effects of various configurations of transmission lines. In an open transmission line ($Z_2 = \infty$) that is initially uncharged (Fig. 4), when the switch is closed, current I_1

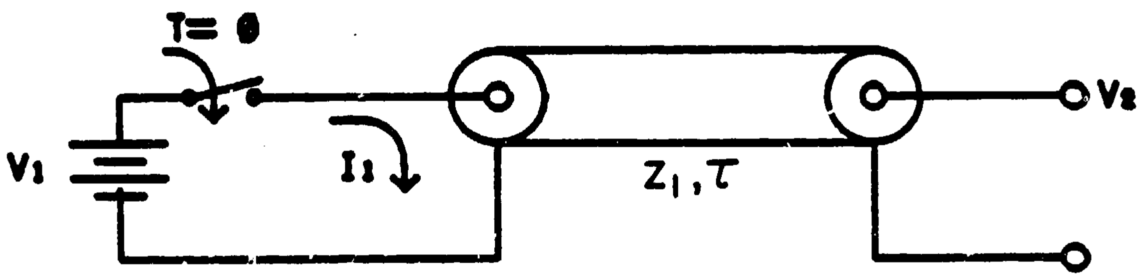
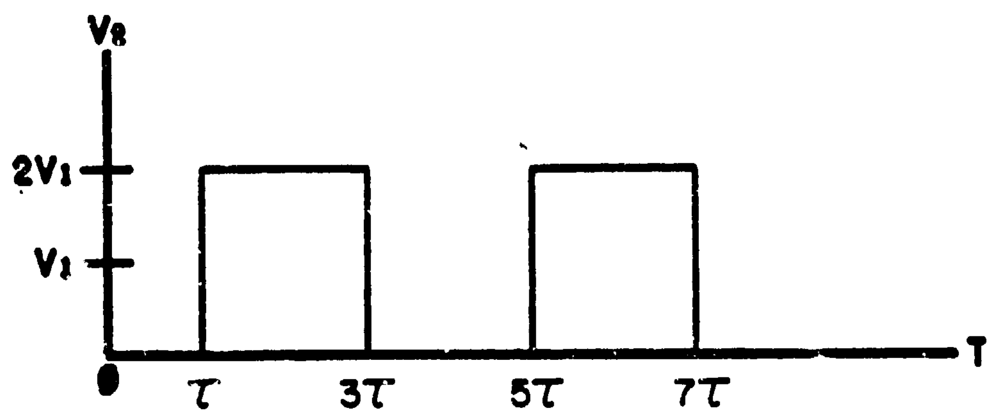


FIG 4: OPEN TRANSMISSION LINE



begins flowing into the line. Current travels down the line until when it reaches time $t = \tau$ (the one-way transmission time of the line) the current must be continuous at the end of the line. Since the line is open there can be no current flowing out of the line, and all the current must reflect and start back down the line in the opposite direction ($I'_1 = -I_1$). Given this situation and equations (10), (11), and (14), the output voltage that results is

$$V_2 = 2\ V_1 \tag{19}$$

or twice the initial voltage. This can sometimes be put to practical use producing a voltage-doubling pulse. These pulses are reflected back down the line and current flows back into the battery. The net result is a voltage pulse of twice the initial amplitude that lasts for two transit times of the line. It is delayed by τ, and it lasts for 2τ. This can be either a help or a nuisance. It is disadvantageous when a fast wave is launched down a transmission line and hits the open end of the line, resulting in voltage doubling, which will track for tremendous distances on a coax cable in air. A resistor ($R >> Z_0$) in series between the voltage source and transmission line prevents this.

Marx banks are complicated by this effect. In Marx banks, there are several biasing resistors associated with each spark gap. If all the resistors and all the capacitors are mounted separately and then connected with a coax cable, pulses traveling back down the coax cables hitting the few-megohm type resistors at the far end can give rise to monumental tracking problems.

Fig. 5 shows the case of a shorted transmission line. The switch is closed and the terminating impedance is zero, so the current builds up in a stairstep fashion. It goes to V_1/Z_1 and then, at time 2τ, it goes to three, then five, then seven times this value. This effect can be used to build a stairstep current generator.

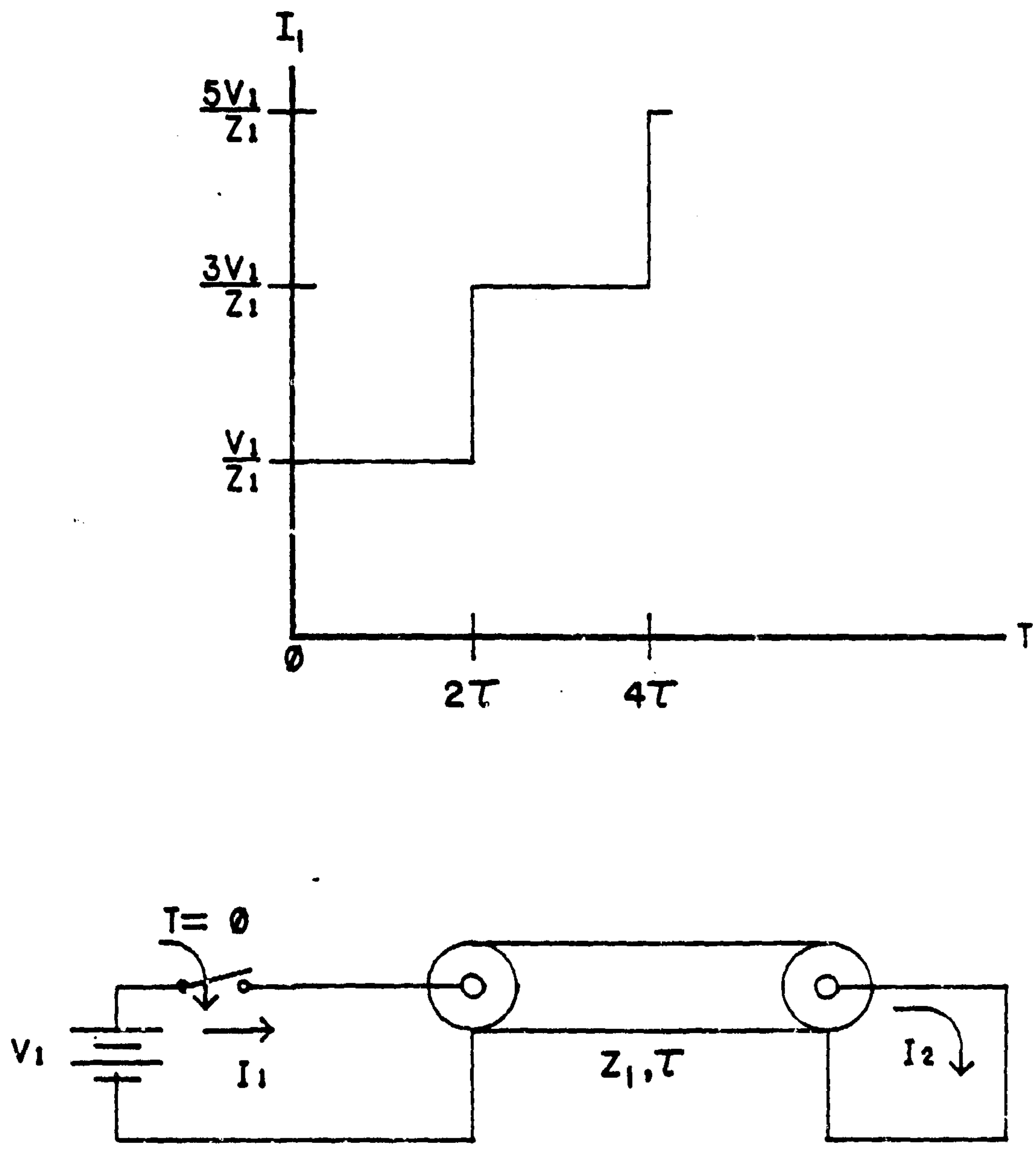


FIG 5: SHORTED TRANSMISSION LINE

Fig. 6 shows a transmission line that is charged to some voltage V_1 and is shorted. The current initially flows in the negative direction. This is a perfect transmission line with no losses. The current in this transmission line will oscillate with a period of 4τ. If a terminating resistor R is inserted and the charged line is switched into this terminating resistor, we have Fig. 7. If the resistance is matched to the impedance in the transmission line, a square pulse of current will result of magnitude

$$I_1 = \frac{-V_1}{R + Z_1} \tag{20}$$

lasting for 2τ.

If the resistance is greater than the impedance of the transmission line, a stairstep generator results where the initial current is found from equation (20). Then, since the energy was not dissipated in a single two-way transit time of the line, it'll stairstep down.

If the terminating impedance is less than the characteristic impedance of the line, the initial current, from equation (20), will have somewhat greater magnitude than in the matched-impedance case, since R is smaller than Z_1, and it wil tend to reverse. Therefore, there is both a current reversal and a voltage reversal in the load, and the circuit will tend to oscillate.

These cases are mostly of academic interest, because in reality there cannot be a perfectly resistive termination. There will always be some inductance in the termination that will round off the leading and trailing edges to make the waveform more sinusoidal. In general, most of the energy eventually ends up in the resistor.

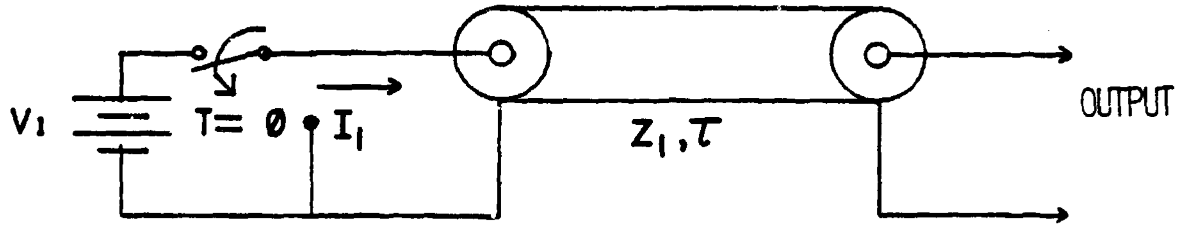
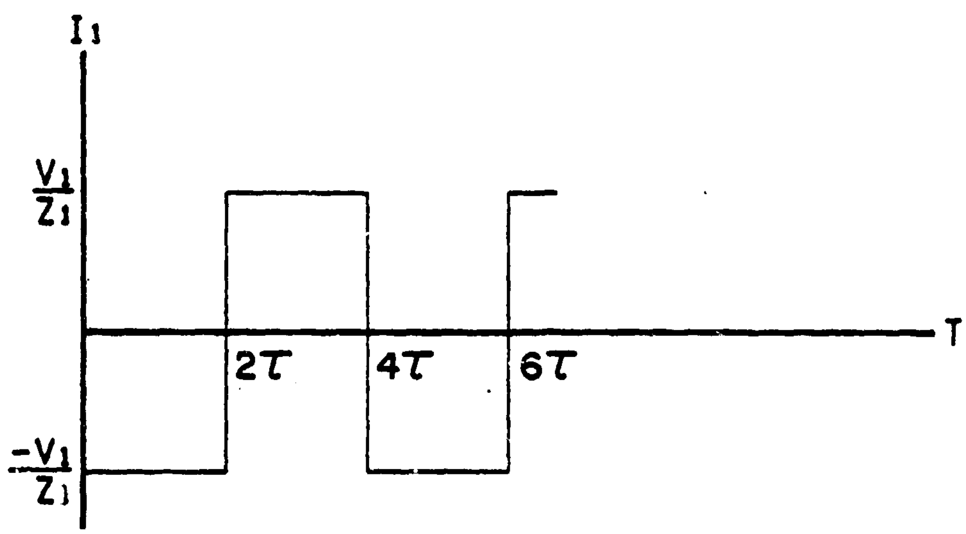
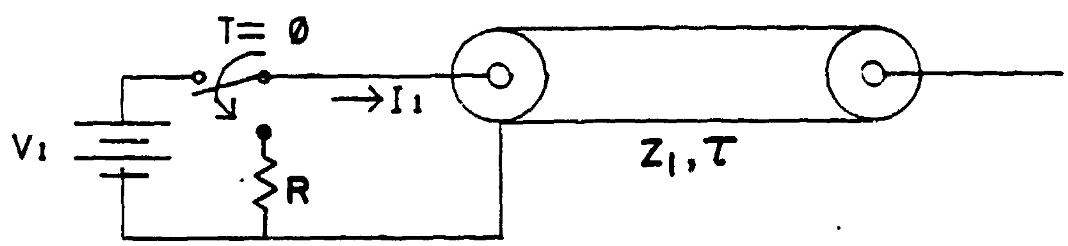


FIG 6: SHORTING OF CHARGED TRANSMISSION LINE





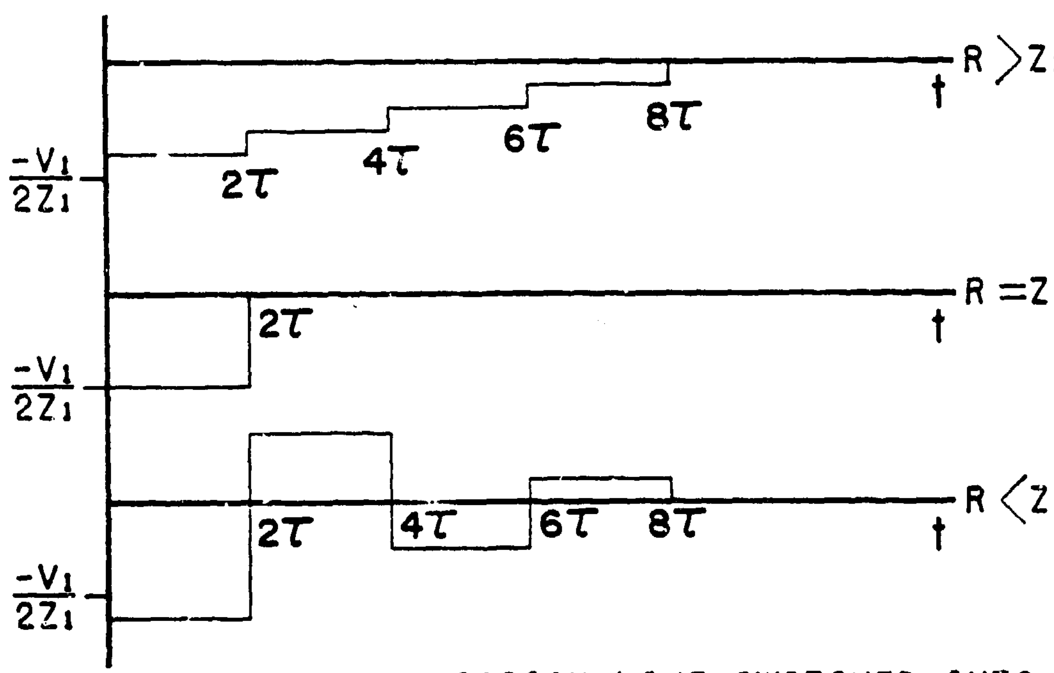


FIG 7: CHARGED TRANSMISSION LINE SWITCHED INTO A LOAD

Also in reality some of the energy is dissipated in the line, although we've been assuming a lossless transmission line. Real transmission lines include a resistive term in series with the line (R_s) (Fig. 8), caused by the resistance in the skin of the line, which can become appreciable at high frequencies. This resistance tends to cause dissipation of the energy traveling down the line.

Another effect, which is usually small, is the parallel conductance or leakage resistance of the dielectric (R_L). Because dielectrics are not perfect they do permit some losses. Usually this is a very large resistance, while the series resistance is fairly small. We shall ignore these in the models: the analysis becomes very complicated if these terms are included. A recent measurement showed a loss of about 1% in a 10-foot length of cable at a frequency of 10 MHz, which is fairly typical of coax cables.

A charged cable loses charge to leakage resistance or corona. If the cable were charged to a high enough voltage to produce corona effects, or if there were voids in the dielectric, corona losses would occur. These may be voltage dependent, and there may be a fairly large resistance up the point where corona begins, and then considerable losses occur. Such corona effects are detrimental to cable life.

Fig. 9 is a case where the transmission line becomes shorted when a battery is switched into an open transmission line. The current oscillates positive and negative with a period 2τ. This current will oscillate in the line, theoretically forever. But the resistive effects previously discussed eventually dissipate that energy, which will settle out at a zero current value and a V_1 voltage value. This current waveform found when a battery is switched into an open transmission line is the same basic current waveform found when shorting a charged transmission line. It appears inverted, but this is a result of the sign convention.

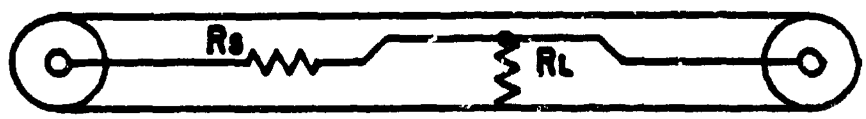


FIG 8: LUMPED LOSS TERMS IN A NON-IDEAL TRANSMISSION LINE

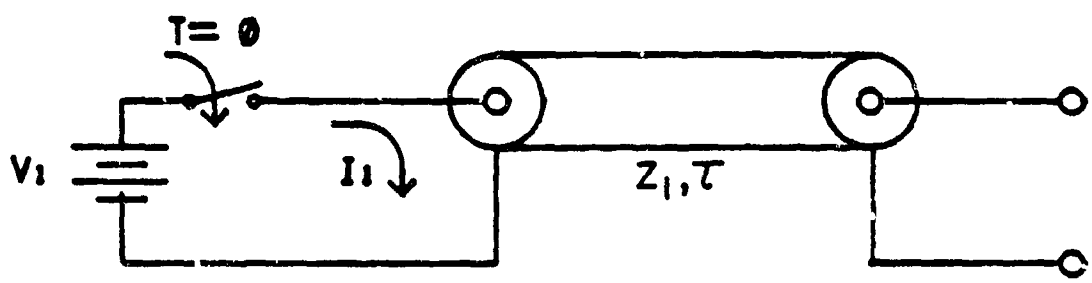
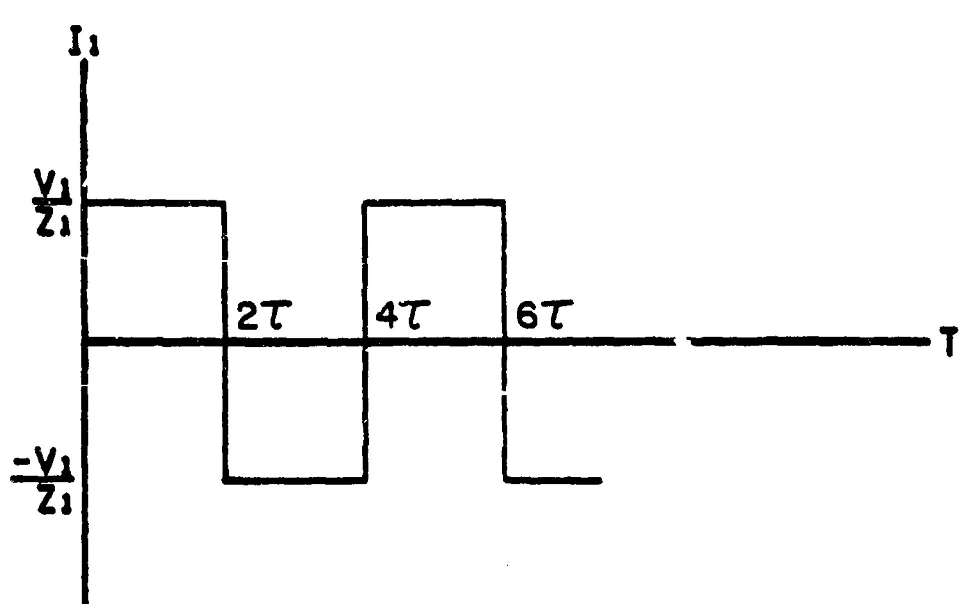


FIG 9: CURRENT IN OPEN TRANSMISSION LINE



This leads to pulse-forming networks; in that if a circuit when shorted produces a current waveform of a particular shape, when the circuit is charged, it returns a current with the same shape when switched into a short circuit or a load resistor. This characteristic allows the design of a pulse-forming network in which the line is replaced with a lumped equivalent.

There are different types of pulse-forming networks. A current-fed line (Fig. 10) is of academic interest, although it is not of much practical use for high-voltage. This can be done with transistors or vacuum tubes to a few hundreds of volts — vacuum tubes permit kilovolt levels. A current source is switched into the shorted line, starting the current flowing into the line. At $t = 0$ the switch is opened. The line will then deliver a pulse into the load resistor. This is only of academic interest because although most switches are able to close under high-voltage conditions, they are difficult to open. It is difficult and yet impractical to build a 50-kV switch that can open quickly. Vacuum interrupters open in a few microseconds or tens of microseconds, but these times are not considered fast. Like most switches, they prefer closing to opening.

This leads to the voltage-fed line, which is charged with some voltage V_O (Fig. 11). (Please notice the designation change.) The transmission line impedance is Z_O, the one-way length is τ, and the voltage is V_O. A closing switch switches into a resistive load at $t = 0$, producing a square pulse. The pulse will be similar to that of a matched load if the load resistance matches the impedance of the transmission line (i.e. the voltage pulse will have a magnitude $V_O/2$ and width of 2τ.

Fig. 12 shows how a lumped equivalent transmission line can be derived from series inductance and parallel capacitance. If the line is continued to the limit at which N approaches infinity and each of the inductances and the capacitances is very small, it looks like a true transmission line with distributed elements.

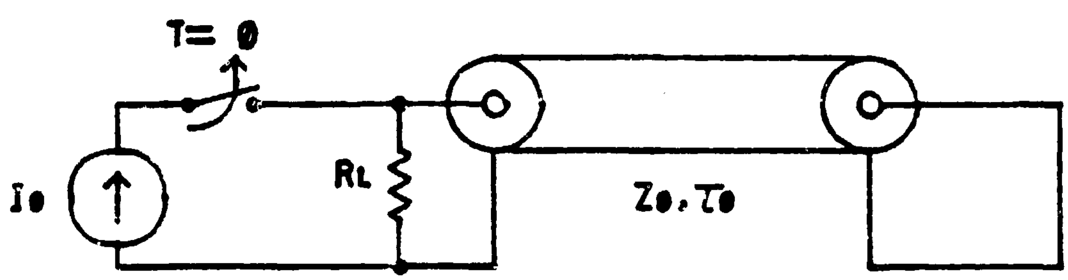


FIG 10: CURRENT FED LINE

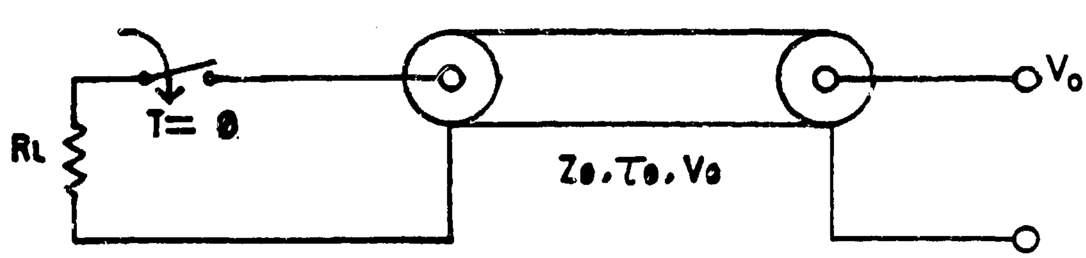


FIG 11: VOLTAGE FED LINE

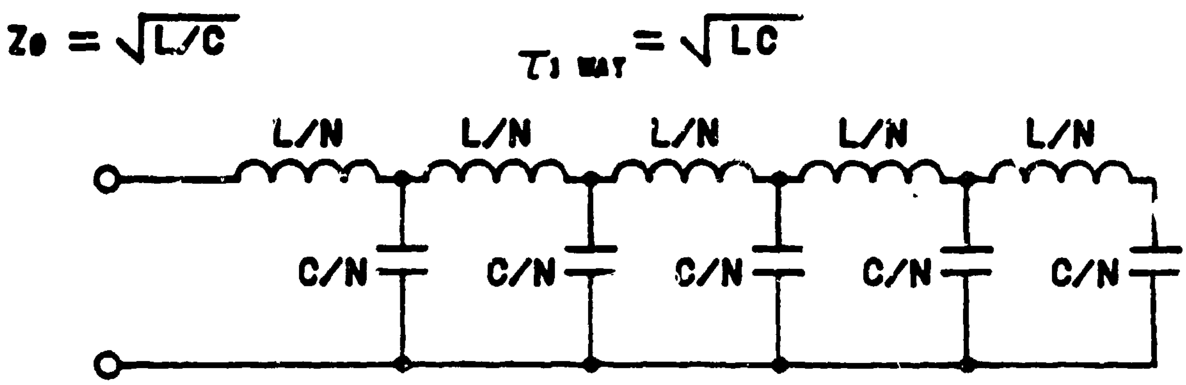


FIG 12: N-SECTION LUMPED EQUIVALENT LINE

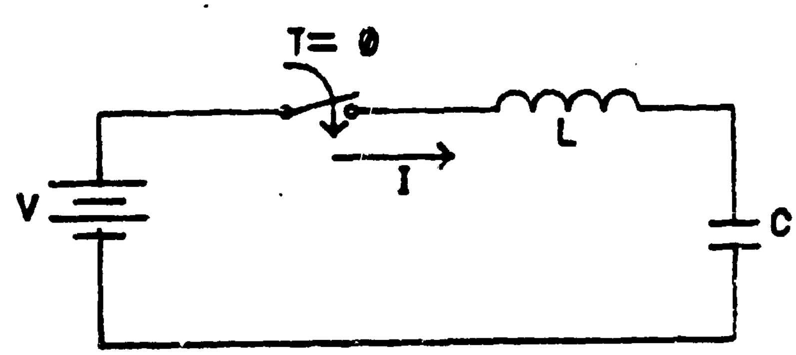


FIG 13: SERIES L-C CIRCUIT

Again, the impedance of the line is the square root of L/C, and this can be either the total series inductance divided by the total parallel capacitance or L_N/C_N. Similarly, the one-way transit time of the line will be the square root of the product of LC: the total series L times the total parallel C.

A Fourier series approximation for a square current waveform may be written as:

$$I(t) = I_m \left[\sin \omega t + \frac{1}{3} \sin 3\omega t + \frac{1}{5} \sin 5\omega t + \ldots\right] \tag{21}$$

This series has to extend to infinity to produce a true square pulse. The amplitudes of the higher harmonics diminish quite rapidly and the line is approximated by some finite series for pulse-forming networks, as it is awkward to have an infinite number of capacitors and inductors. If a battery is switched into a series inductance and capacitance (Fig. 13), the current will have the form:

$$I(t) = \frac{V_o}{Z_o} \sin\omega t \tag{22}$$

where Z_o is:

$$Z_o = \sqrt{\frac{L}{C}} \tag{23}$$

and ω is:

$$\omega = \frac{1}{\sqrt{LC}} = \frac{\pi}{\tau} \tag{24}$$

In the case of a one-way transit time τ, ω appears as a $1/\tau$ term, which leads to Guillemin networks.

The Type-C Guillemin network (Fig. 14) is our first item of interest. The battery is switched into a five-stage network, numbered C1, C3, C5, C7, and C9. These stages are numbered to produce odd harmonics (e.g. the "third harmonic" is produced from the L3-C3 combination). For current oscillation as in a cable PFN, the proportionality is written:

$$I(t) \propto \left[\frac{1}{Z_1} \sin \omega_1 t + \frac{1}{Z_3} \sin \omega_3 t + \frac{1}{Z_5} \sin \omega_5 t + \frac{1}{Z_7} \sin \omega_7 t + \frac{1}{Z_9} \sin \omega_9 t \right] \tag{25}$$

You will notice that the I_{max} term has been temporarily deleted. Comparing terms with equations (21) and (25), it is evident that the conditions necessary for square-wave oscillations are:

$$Z_1 = 1, \quad Z_3 = 3, \quad Z_5 = 5, \quad Z_7 = 7, \quad Z_9 = 9,$$

$$\omega_1 = \omega, \quad \omega_3 = 3\omega, \quad \omega_5 = 5\omega, \quad \omega_7 = 7\omega, \text{ and } \omega_9 = 9\omega.$$

By comparison to equations (23) and (24), we can generalize:

$$Z_n = n\sqrt{\frac{L_1}{C_1}} = \sqrt{\frac{L_n}{C_n}} \tag{26}$$

and

$$\omega_n = \frac{1}{\sqrt{L_n C_n}} \tag{27}$$

PFN's

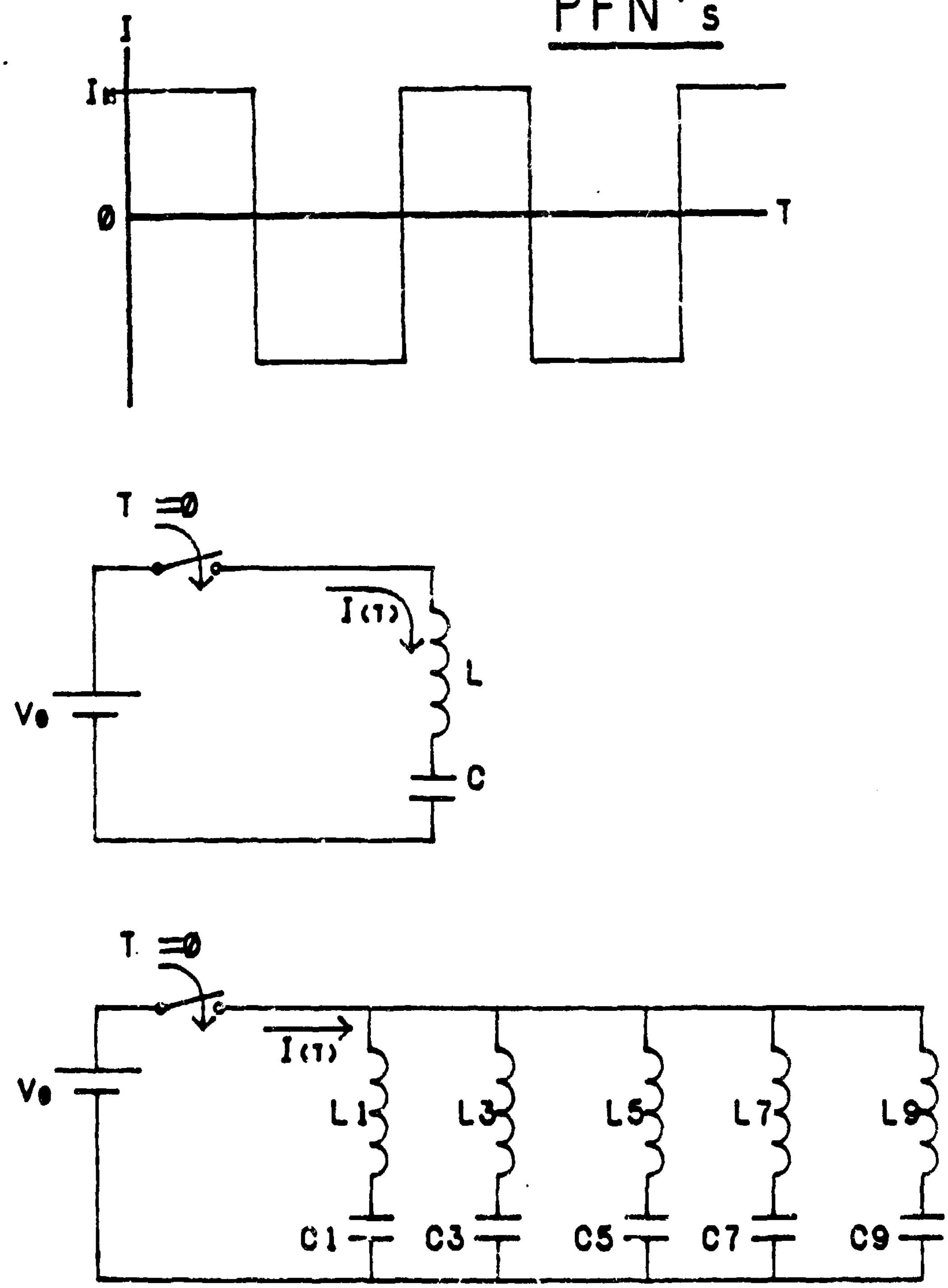


FIG 14: 5-STAGE TYPE C GUILLEMIN NETWORK

We can now find the nth term by comparison to Equation (22) and by noting that all the impedances (Z_n) are in parallel:

$$Z_0 = Z_1 \,||\, Z_3 \,||\, Z_5 \,||\, Z_7 \,||\, Z_9 \tag{28}$$

One way of solving for the values in this circuit is to assume the capacitors are all charged to V_o and the circuit is switched into a load resistor of value $R = Z_o$ (Fig. 15). By analogy to the transmission line model, the current could be assumed to be a square pulse of amplitude of $V_o/2Z_o$ and of a duration τ . τ now is used as the <u>full width</u> of the pulse instead of the half width. This is done to be consistent with the text.[3] If there is matched impedance ($R = Z_o$), then all the energy stored in the PFN will be deposited in R during the pulse. This energy can be calculated from the definite integral of the power:

$$w = i^2R = \left(\frac{V_o}{2Z_o}\right)^2 Z_o = \frac{V_o^2}{4Z_o} \tag{29}$$

which is written

$$E = \int_o^{\tau} w\, dt = \frac{V_o^2 \tau}{4Z_o} \tag{30}$$

This energy may be equated to the energy stored on the total capacitance of the PFN

$$E = \frac{1}{2} C_T V_o^2 \tag{31}$$

and solve for the total capacitance

$$C_T = \frac{\tau}{2Z_o} \tag{32}$$

where Z_0 is the load resistance in which the energy is deposited. In this type of network all the inductances are equal.

The difference in τ is visible in the capacitances of the coax lines. When τ is the one-way transit time, the capacitance of a coax or a regular transmission line is simply τ/Z_0.

Because there is not an infinite number of elements in this Fourier series there will not be a perfectly square pulse, and the energy deposited will be slightly off because it is not a square pulse. In a five-stage pulse-forming network, the energy will be about 4% off. If there are fewer stages, the discrepancy will be somewhat greater.

The pulse width for this case can also be calculated from

$$\tau = \frac{\pi}{\omega_1} \tag{33}$$

this is from the fact that

$$\omega_1 = 2\pi f_1 \tag{34}$$

The period of the fundamental frequency is

$$T = \frac{1}{f_1} = \frac{2\pi}{\omega_1} \tag{35}$$

The pulse width (τ) is the half period of the fundamental frequency, and the pulse width from the PFN will be

$$\tau = \frac{T}{2} \tag{36}$$

Rearranging Equation (33) results in

$$\omega_1 = \frac{\pi}{\tau} \tag{37}$$

The inductance may be calculated from

$$L_n = L = \frac{1}{\omega_1^2 C_1} = \frac{1}{(\pi/\tau)^2 \; C_1} = \frac{\tau^2}{\pi^2 C_1} \tag{38}$$

From Equation (26) we can find

$$C_n = \frac{C_1}{n^2} \tag{39}$$

which puts the capacitors in a ratio of 1:1/9:1/25:1/49:1/81. Therefore:

$$C_T = C_1 + \frac{C_1}{9} + \frac{C_1}{25} + \frac{C_1}{49} + \frac{C_1}{81} = 1.184\ C_1 \tag{40}$$

$$C_1 = \frac{C_T}{1.184} = \frac{\tau}{2.368Z_o} \tag{41}$$

From Equations (38) and (41):

$$L = \frac{2.368Z_o\tau}{\pi^2} = 0.24Z_o\tau \tag{42}$$

The remaining capacitors are found from Equations (39) and (41). Fig. 16 shows a typical waveform for this type of circuit, done on the digital computer. It was calculated to produce a 1-microsecond pulse width with an amplitude of 10,000 A. The PFN is initially charged to 20,000 volts, and it has a 1-ohm impedance and is switched into a 1-ohm resistor, so it should have about a 10-kA current for a microsecond. There is quite a bit of overshoot from the leading edge and the trailing edge because

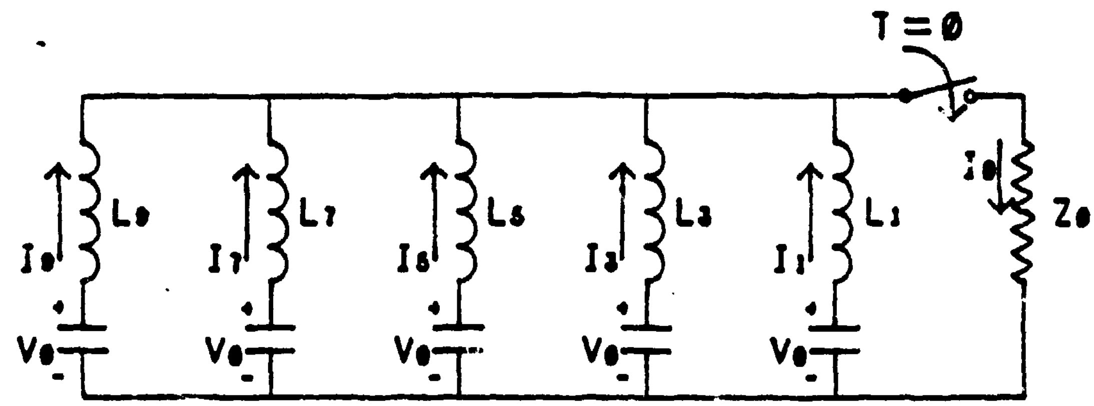


FIG 15: TYPE C GUILLEMIN NETWORK WITH MATCHED LOAD

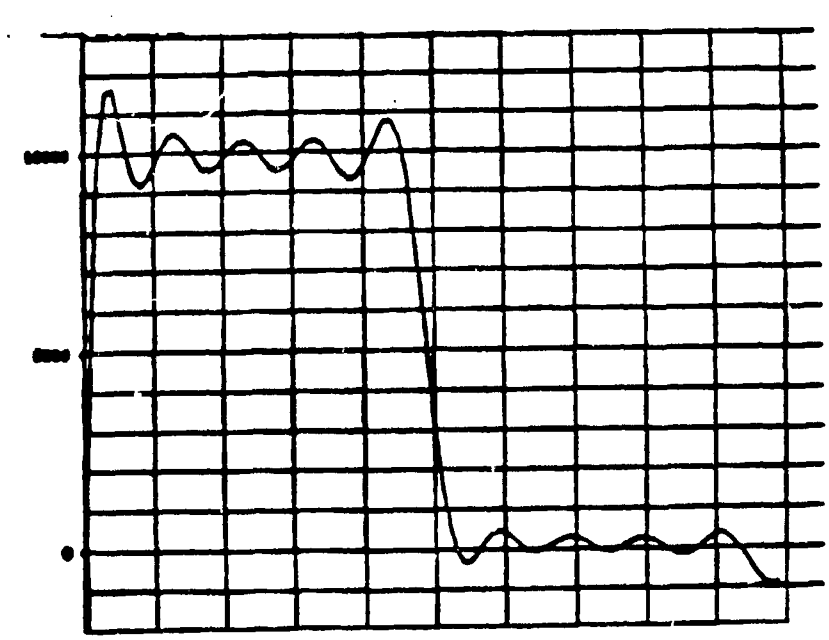


FIG. 16. TYPE C WAVEFORM

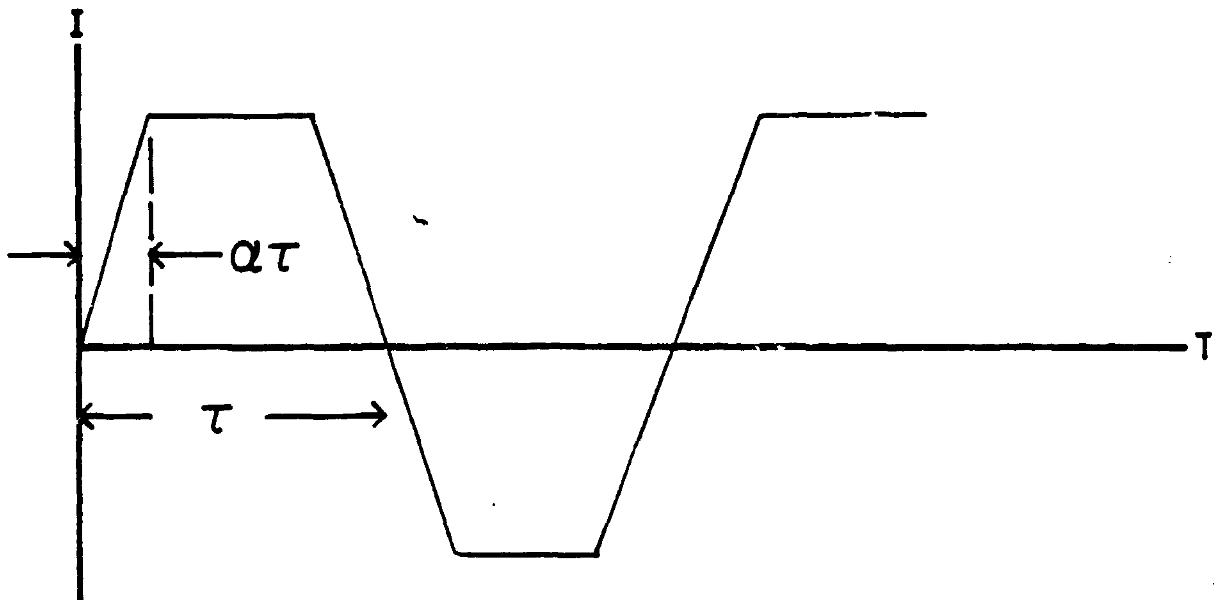
the eleventh harmonic is not there to reduce it. The ringing diminishes somewhat toward the middle of the pulse. As more stages are added to the pulse-forming network, there is less overshoot on corners and less ringing in the middle.

This effect led Guillemin to write the equation for a trapezoid and assume that he had a linear risetime, a flat pulse, and a linear falltime. He made the risetime some percentage (a) of the total pulse length (Fig. 17). With "a" a percentage of the risetime, one can relax the requirements on a pulse-forming network. As the requirements are relaxed the "a" becomes larger, and the number of terms required in the pulse-forming network is reduced. A point is reached where the capacitor for the 9th harmonic becomes either zero or slightly negative, and the inductance becomes slightly negative or infinite and can be deleted. The contribution from those higher order terms are unnecessary to generate the trapezoidal waveforms.

Fig. 18 shows another case examined by Guillemin: a flat-topped waveform with parabolic rise- and falltime. The total risetime is used as some percentage of the total pulse width (a) and the only difference is the parabolic rise- and falltimes.

Fig. 19 shows some computer plots of the waveforms for the five-stage trapezoid and the five-stage parabolic generators terminated in the correct impedance. These waveforms are not ideal — in fact they look similar to Fig. 18. The main difference is that the overshoot is less. The few irregularities in the middle of the pulse are not exactly on the first, third, fifth, seventh, and ninth harmonics, but on some slight variation.

As one approaches the 10% trapezoidal waveform, the pulse becomes smoother and as he approaches the 10% parabolic, it becomes lumpier. These are still better than the rectangular pulse in Fig. 18. Although these are meant to be the same pulse width, this is difficult to compare because they are plotted on graphs of different scales. However, the



IG 17: TRAPEZOIDAL WAVEFORM FLAT TOP WITH PARABOLIC RISE & FALL

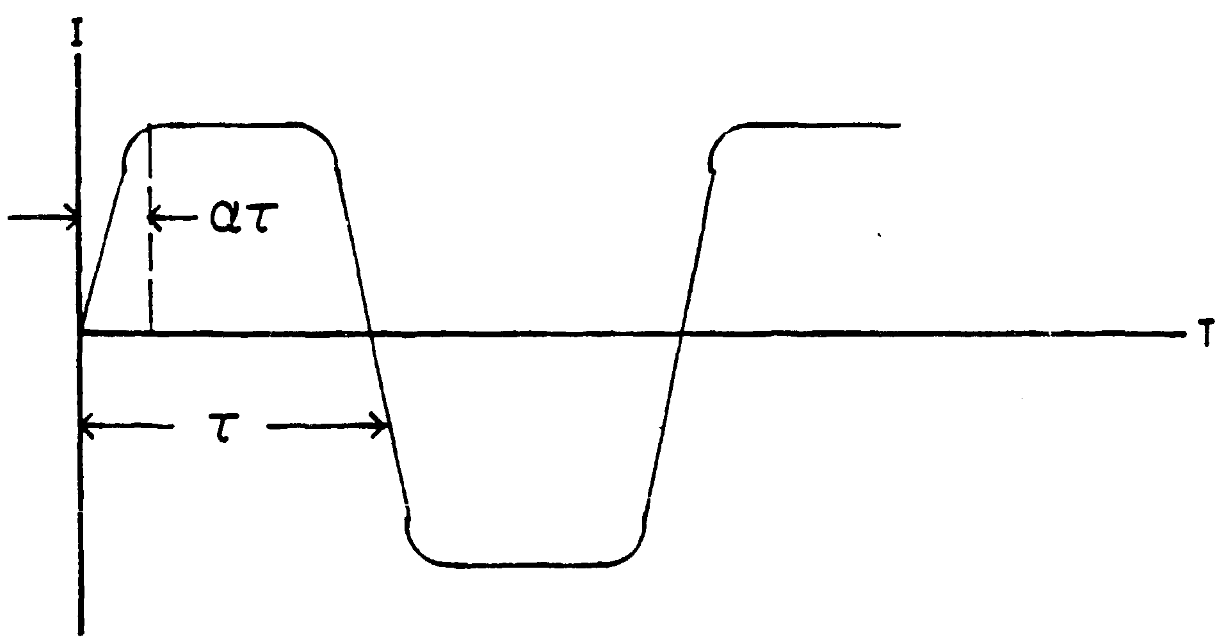
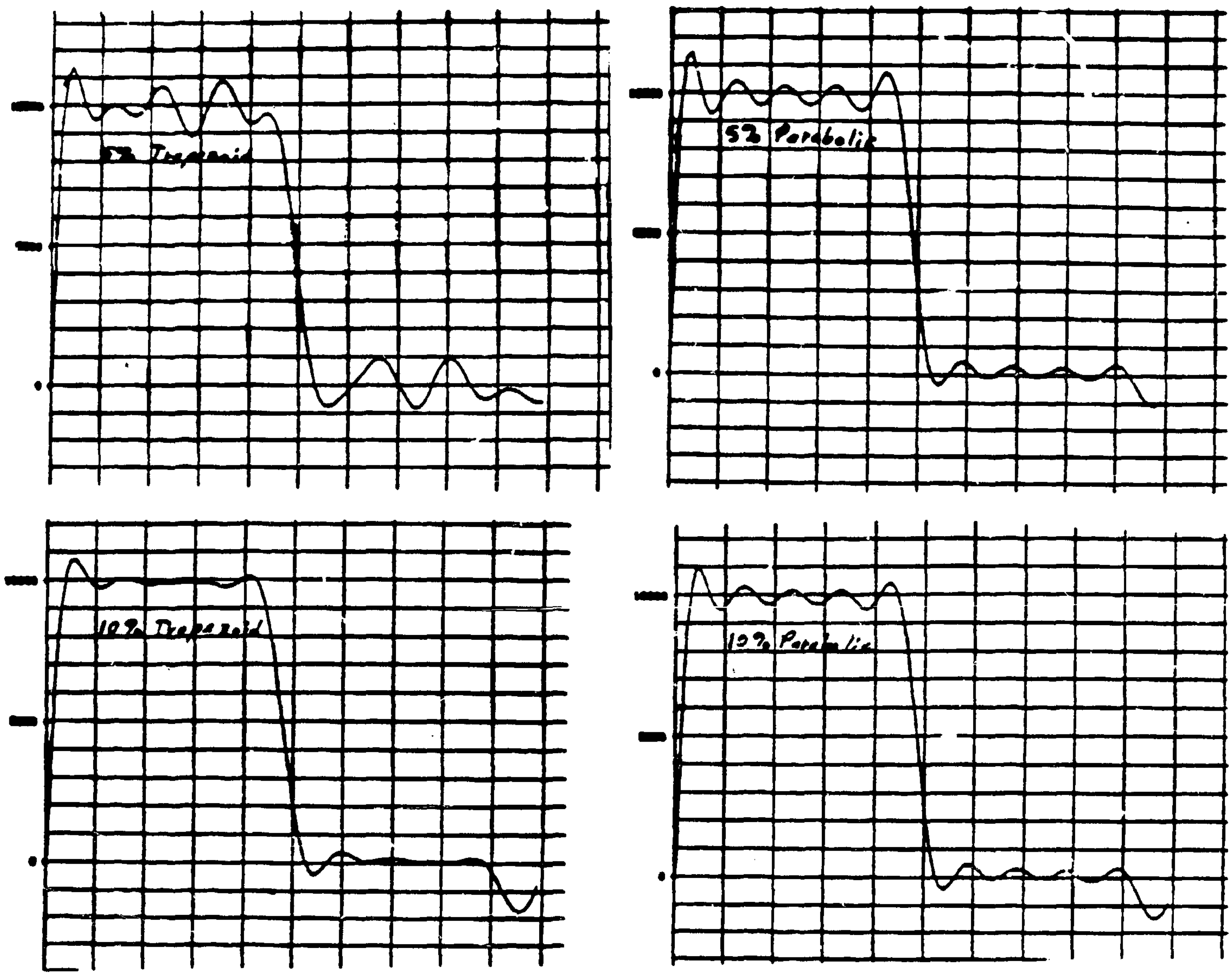


FIG 18: PARABOLIC RISE AND FALL TIME



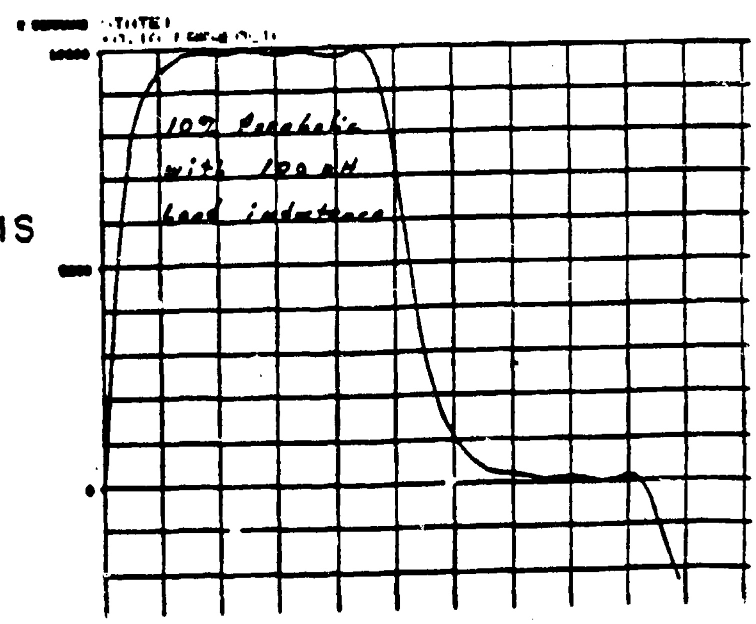


FIGURE 19. COMPUTER WAVEFORMS

risetime is obviously good. A 100-nH inductance put in series at the load effectively increases the last inductor by 100 nH, producing an L/R risetime of 100 ns, which is a 10% value for the parabolic load. It rounds off the leading edge of the waveform, flattens most of the ripples, and rounds off the trailing edge. For the most part, on all of these there is very little energy deposited in the resistance after the main pulse.

Guillemin found other circuits that produce the same general waveform (Fig. 20). Type A has parallel LC resonant circuits with one capacitor and one inductor in series.

All these circuits have contained capacitors of different values. Type B also has various inductances and capacitances, although it looks more like a classic transmission line. This configuration is requisite to generate the trapezoidal pulse with a nominal 8% risetime.

The Type F has all the capacitors in series in a transposition of the Type B network, replacing all the capacitors with inductors, etc. Unfortunately, these capacitors have some very unusual values.

The Type D network uses capacitors of equal value, but has negative values of inductance in series with several of the capacitors. Coupling between inductors can realize this physically and produce the Type E Guillemin network. In this network, there is a coefficient of coupling between the various inductors and all the capacitors are of equal value.

Total inductances desired to produce these mutual inductances can be found from:

$$L_T = \frac{\tau Z_\Omega}{2} \tag{43}$$

and the capacitances are derived from

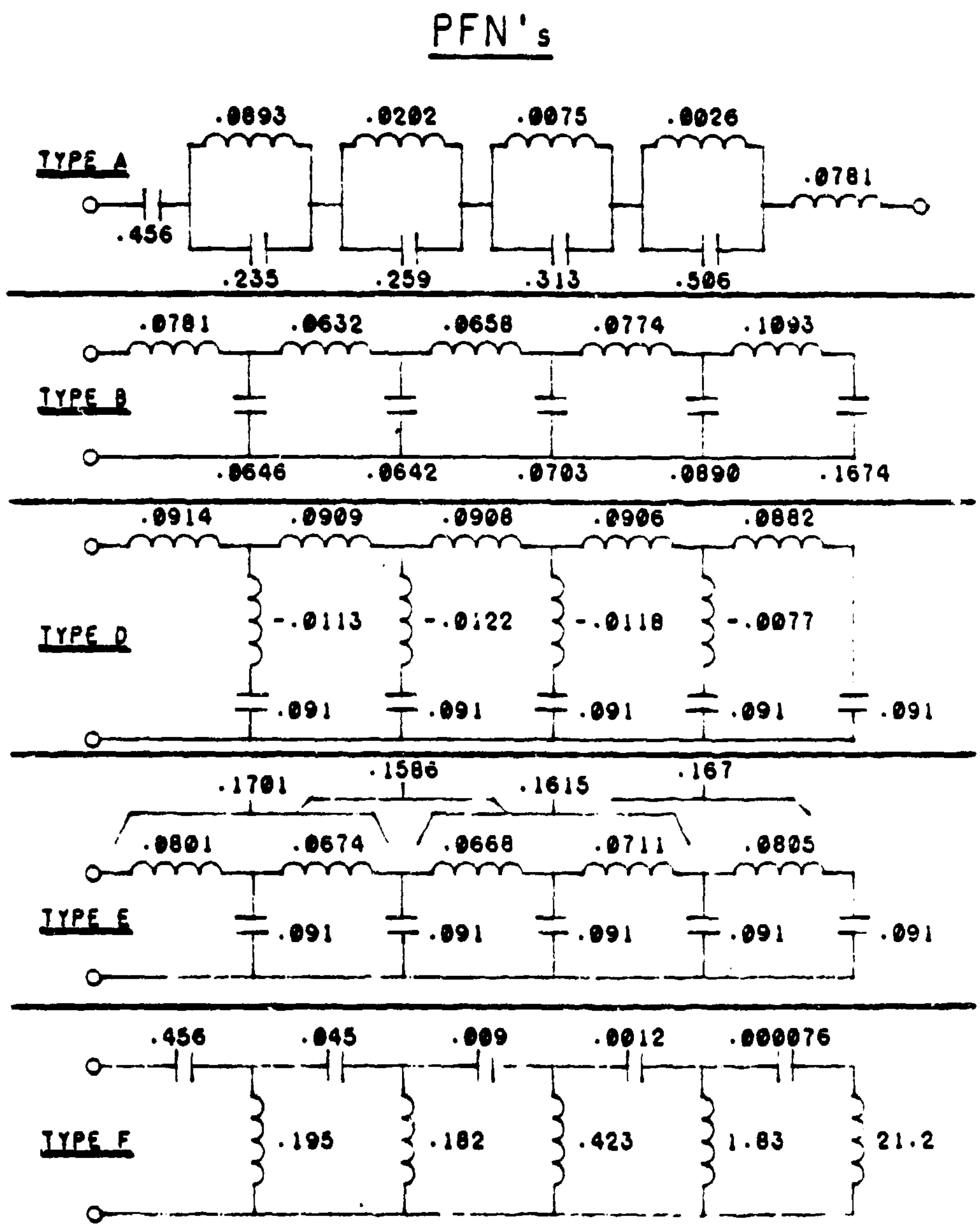


FIG 20: OTHER GUILLEMIN NETWORKS

$$C_T = \frac{\tau}{2Z_o} \qquad (44)$$

This capacitance is divided equally among the number of stages. Then a coil is wound on a long cylindrical form having the total desired inductance, with 20%-30% additional inductance added to the coils on each end of this form. The ratio of length to diameter should be chosen to provide mutual inductance of about 15% of the self-inductance in each center section. Soldering the capacitors on results in the correct diameter-to-length ratio, winds the coil, and taps it at the appropriate number of turns.

There are certain minimum inductance limitations in the various types of networks. Some PFNs have inductors in series with capacitors; others, like the Type C network with its coil wound on a form, require a somewhat larger minimum inductance value. The importance of this minimum inductance number can be seen from rearranging Equation (43) to calculate a minimum pulse width-impedance:

$$\frac{\tau Z_o}{2} \geq L_{min} \qquad (45)$$

On a Type A network, if the minimum inductance is reduced to 100 nH, then the Z_o product must be greater than or equal to 38-1/2 times 10^6. Since the Type E network has mutual inductance, the absolute minimum inductance is about a microHenry, which produces a 1 x 10^6 τZ_o product. Type C networks may have minimum inductances to the level found in the capacitors plus the connections.

Capacitors may be bought of almost any inductance, but vary greatly. A practical lower limit does exist for capacitors. Maxwell Series S capacitors have about 20 nH in the capacitor and can usually be connected into a circuit in which the inductance of connections can be kept down to 20 nH. An inductance ot 40 nH produces a τZ_0 of 0.16 microseconds - ohms.

This is plotted on a graph in Fig. 21. If the desired pulse width and impedance of load are known, this circuit will provide some idea of the practical minimums available in Guillemin-type networks. Notice that at impedances of one ohm, pulse width remains around a microsecond with a Type C Guillemin, but that generally low impedances dictate long pulse widths. High impedances allow short pulse widths. Ten ns is a reasonable minimum pulse width.

Also available on the graph is the useful range of distributed pulse-forming networks (coaxial cables). With enough coaxial cables in parallel, impedance can be reduced to 1 ohm. Because coax cables with a polyethylene dielectric have a one-way propagaation time of about a nanosecond per foot, given a fast enough switch, a 1-ns pulse width can be attained on a cable or transmission line. Strip lines provide low impedances but are difficult to switch. Neither cable nor lumped PFNs will generally deliver the desired pulse unless the switch closes much faster than the desired pulse width. Switching, therefore, presents a problem.

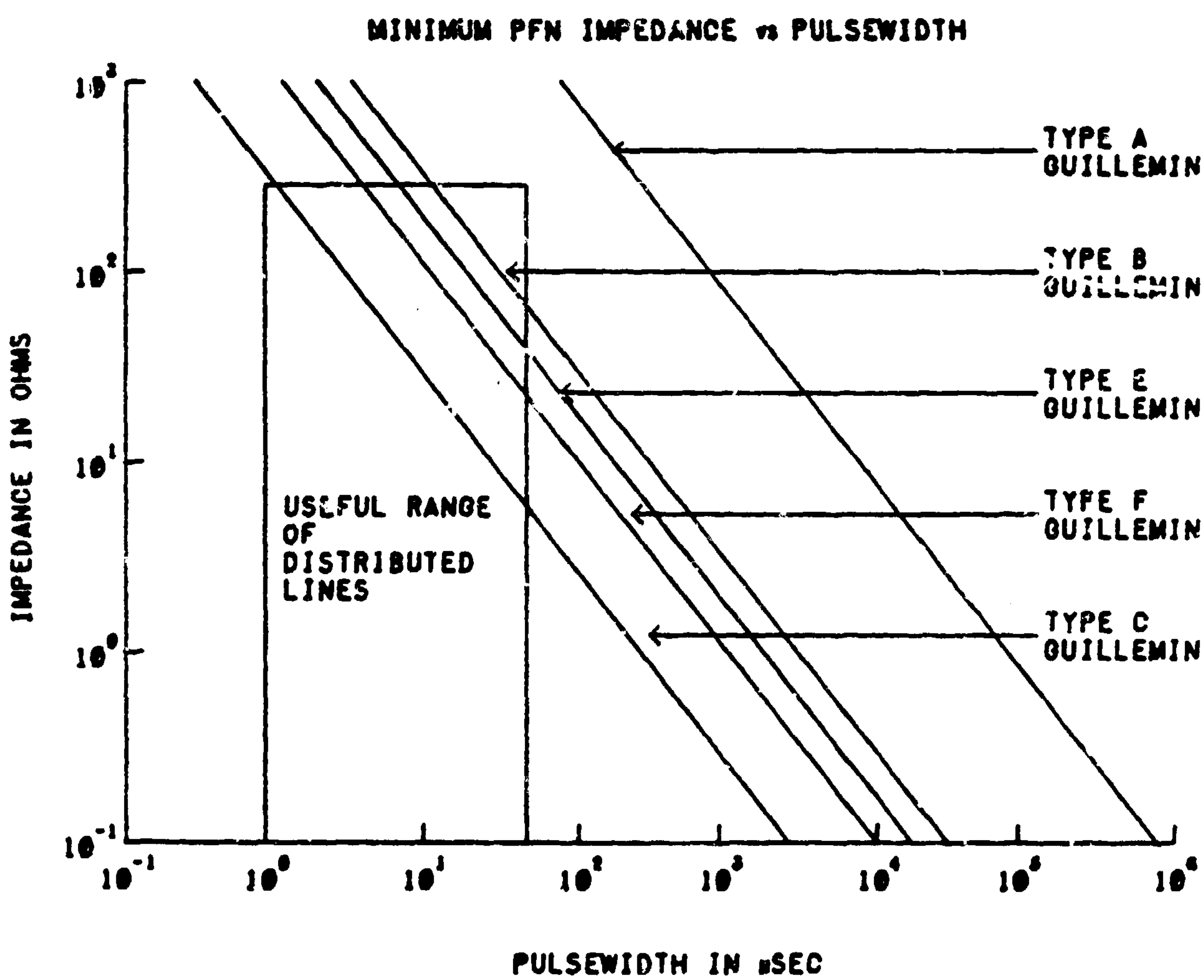


FIG 21: PFN PARAMETERS DETERMINED BY MINIMUM ATTAINABLE INDUCTANCE

1. E. G. Cook and T. R. Burkes, Pulse Forming Network Investigation, Dept. of Electrical Engineering, Texas Tech University, Lubbock, Texas, August 1975.

2. D. G. Ball and T. R. Burkes, Pulse Generation for Time-Varying Loads, Dept. of Electrical Engineering, Texas Tech University, Lubbock, Texas, August 1975.

3. G. N. Glasoe and J. V. Lebacqz, Pulse Generators, Dover Publications, Inc., Chapter 6.